

# HY-PDE workshop 2021

## May 26th (Wednesday)

- 10:00 - 10:30 고동남 교수 (가톨릭대)

Title: Uniform error estimates for random batch methods to first-order consensus models.

Abstract: Random Batch Methods (RBM) was proposed and analyzed recently for a general interacting particle systems in order to approximate the time-evolution of solutions. The idea of RBM originated from the (mini-batch) stochastic gradient methods, and hence its error  $l^2$ -expectation error is naturally bounded in a finite time. Moreover, for contracting dynamics, the error is bounded uniformly on time. In this study, we extend uniform error estimates to the first-order consensus model, which has heterogeneous intrinsic dynamics and convolution-type consensus interactions, as an example, the Kuramoto model. Under a priori assumption on the knowledge of equilibrium point, we suggest a refined RBM to the first-order consensus model to get uniform error estimates in expectation and almost-sure sense.

- 10:35 - 11:05 최경수 교수 (KIAS)

Title: Ancient gradient flows of elliptic functionals

Abstract: We will discuss about ancient solutions to a class of parabolic PDEs which are gradient flows of elliptic functionals. When an ancient solution is asymptotic to a self-similar solution, we can use the spectrum of the linearized operator over the limit self-similar solution to classify ancient solutions. In time permit, we will talk about its applications to the mean curvature flows.

- 11:10 - 11:40 정인지 교수 (서울대)

Title: Filamentation near Hill's vortex

Abstract: For the axi-symmetric incompressible Euler equations, we prove linear in time filamentation near Hill's vortex: there exists an arbitrary small outward perturbation growing linearly for all times. This is based on combining recent nonlinear stability statements with a dynamical bootstrapping scheme for particle trajectories. These results confirm numerical simulations by Pozrikidis (1986). Joint work with Kyudong Choi (UNIST).

- 13:30 - 14:00 배한택 교수 (UNIST)

Title: The formation of singularities for some water wave models

Abstract: In this talk, we show how some water wave models can develop singularities in finite time for a certain class of smooth initial data. The models covered in this talk are two inviscid models (abcd-Boussinesq equations and Green-Naghdi equations) and one viscous model (Kakutani-Matsuuchi equation).

- 14:05 - 14:35 이태훈 박사 (KIAS)

Title: Gauss curvature flow with an obstacle

Abstract: We consider the obstacle problem for the Gauss curvature flow with an exponent  $\alpha$ . Under the assumption that both the obstacle and the initial hypersurface are strictly convex closed hypersurfaces and that the obstacle is enclosed by the initial hypersurface, uniform estimates are obtained for several curvatures via a penalty method. We also prove that when the hypersurface is two dimensional with  $0 < \alpha \leq 1$ , the solution of the Gauss curvature flow with an obstacle exists for all time with bounded principal curvatures  $\{\lambda_i\}$ , where the upper bound is uniform, and the lower bound depends on the distance from the free boundary. Moreover, we show that there exists a finite time  $T_*$  after which the solution becomes stationary in the same shape as the obstacle.

- 14:55 - 15:25 최범준 박사(KIAS)

Title: Liouville theorem for surfaces translating by sub-affine-critical powers of Gauss curvature

Abstract: We construct and classify the translating solutions to the flows by sub-affine-critical powers of the Gauss curvature in  $\mathbb{R}^3$ . If  $\alpha$  denotes the power, this corresponds to a Liouville theorem for degenerate Monge-Ampere equations  $\det D^2u = (1 + |Du|^2)^{2-\frac{1}{2\alpha}}$  on  $\mathbb{R}^2$  for  $0 < \alpha < 1/4$ . For the affine-critical case  $\det D^2u = 1$ , a classical result by Jörgen, Calabi and Pogorelov shows the level curves of given solution are homothetic ellipses. In our case, the level curves converge asymptotically to a round circle or a curve with  $k$ -fold symmetry for some  $3 \leq k \leq n_\alpha$ . More precisely, these curves are closed shrinking curves to the  $\frac{\alpha}{1-\alpha}$ -curve shortening flow that were previously classified by Andrews in 2003. This is a joint work with Kyeongso Choi and Soojung Kim.

- 15:30 - 16:00 배기찬 박사(서울대)

Title: The relativistic quantum Boltzmann equation near equilibrium

Abstract: The relativistic quantum Boltzmann equation (or the relativistic Uehling-Uhlenbeck equation) describes the dynamics of single-species fast-moving quantum particles. With the recent development of the relativistic quantum mechanics, the relativistic quantum Boltzmann equation has been widely used in physics and engineering such as in the quantum collision experiments and the simulations of electrons in graphene. In spite of such importance, there has been no mathematical theory on the existence of solutions for the relativistic quantum Boltzmann equation to the best of authors' knowledge. In this talk, we consider the global existence of a unique classical solution to the relativistic Boltzmann equation for both bosons and fermions when the initial distribution is nearby a global equilibrium.

## May 27th (Thursday)

- 10:00 - 10:30 정진욱 박사 (서울대)

Title: Asymptotic analysis for a Vlasov-Fokker-Planck/Navier-Stokes system in a bounded domain

Abstract: We study an asymptotic analysis of a coupled system of kinetic and fluid equations. More precisely, we deal with the nonlinear Vlasov-Fokker-Planck equation coupled with the compressible isentropic Navier-Stokes system through a drag force in a bounded domain with the specular reflection boundary condition for the kinetic equation and homogeneous Dirichlet boundary condition for the fluid system. We establish a rigorous hydrodynamic limit corresponding to strong noise and local alignment force. The limiting system is a type of two-phase fluid model consisting of the isothermal Euler system and the compressible Navier-Stokes system. Our main strategy relies on the relative entropy argument based on the weak-strong uniqueness principle. For this, we provide a global-in-time existence of weak solutions for the coupled kinetic-fluid system. We also show the existence and uniqueness of strong solutions to the limiting system in a bounded domain with the kinematic boundary condition for the Euler system and Dirichlet boundary condition for the Navier-Stokes system.

- 10:35 - 11:05 윤석배 교수(성균관대)

Title: Shakov model near equilibrium

Abstract: In this talk, we address the existence and asymptotic stability of the Shakhov model when the initial data is a small perturbation of a global equilibrium. We observe that the linearized relaxation operator becomes more degenerate when the Prandtl number vanishes, and the micro-macro equation that involves a non-conservative quantity related to the heat flux has to be considered to derive the full coercivity. This is joint work with Gi-Chan Bae.

- 11:10 - 11:40 강문진 교수 (KAIST)

Title: Stability of a composite wave of viscous shock and rarefaction for isentropic Navier-Stokes equations

Abstract: I will present a recent result on time-asymptotic stability of solutions to the compressible Navier-Stokes equations (1D isentropic case) with initial data connecting two different constant states, which are imposed by Riemann data generating a composite wave of viscous shock and rarefaction wave. The main idea of the proof is to use the so called weighted relative entropy method with shift instead of the (typical) anti-derivative method used for the stability of shocks, which does not match well with the energy method used for the stability of rarefactions.

- 13:30 - 14:00 최종근 교수 (부산대)

Title: Mixed Dirichlet-conormal boundary value problems for parabolic operators

Abstract: I will present maximal regularity for parabolic equations in divergence form with mixed boundary conditions in a cylindrical domain with a time-dependent separation. Such mixed boundary conditions arise naturally in free boundary problems, for instance, when a block of floating ice is melting into liquid water, the ice-water interface maintains zero temperature (Dirichlet) while the ice-air interface is insulated (conormal). This is joint work with Hongjie Dong(Brown, USA) and Zongyuan Li(Rutgers, USA).

- 14:05 - 14:35 김도현 교수 (성신여대)

Title: Formation of phase-locked states for a coupled system of Schrodinger equations

Abstract: In this talk, we introduce a coupled system of Schrodinger equations, so-called the Schrodinger-Lohe model as a phenomenological model for synchronization which occurs in a quantum regime. When the external one-body potentials are given to be different, all wavefunctions would not collapse to one wavefunction due to their heterogeneity. Instead, relative distances between wavefunctions tend to positive definite values provided that a coupling strength is sufficiently large; in other words, phase-locked states are formed. For a small coupling strength, periodic motion might be observed.

- 14:55 - 15:25 최영필 교수 (연세대)

Title: Quantified overdamped limit for Vlasov-Fokker-Planck equations with singular interaction forces

Abstract: In this talk, I will discuss a quantified overdamped limit for kinetic Vlasov-Fokker-Planck equations with nonlocal interaction forces. We provide explicit bounds on the error between solutions of that kinetic equation and the limiting equation, which is known under the names of aggregation-diffusion equation or McKean-Vlasov equation. Our strategy only requires weak integrability of the interaction potentials, thus in particular it includes the quantified overdamped limit of the kinetic Vlasov-Poisson-Fokker-Planck system to the aggregation-diffusion equation with either repulsive electrostatic or attractive gravitational interactions.

- 15:30 - 16:00 심우주 박사 (서울대)

Title: A mean-field limit of the Cucker-Smale model on complete Riemannian manifolds

Abstract: We study a mean-field limit of the Cucker-Smale(C-S) model for flocking on complete smooth Riemannian manifolds. For this, we first formally derive the kinetic manifold C-S model on manifolds using the BBGKY hierarchy and derive several a priori estimates on emergent dynamics. Then, we present a rigorous mean-field limit from the particle model to the corresponding kinetic model by using the generalized particle-in-cell method. As a byproduct of our rigorous mean-field limit estimate, we also establish a global existence of a measure-valued solution for the derived kinetic model. Compared to the corresponding results on  $\mathbb{R}^d$ , our procedure requires additional assumption on holonomy and proper *a priori* bound on the derivative of parallel transports. As a concrete example, we verify that hyperbolic space  $\mathbb{H}^d$  satisfies our proposed standing assumptions.

**May 28th (Friday)**

- 10:00 - 10:30 김유찬 교수 (서울시립대)

Title: Piecewise smoothness for linear elliptic systems with piecewise smooth coefficients

Abstract: "In this talk, we obtain piecewise  $C^{m+1,\gamma}$  regularity for linear elliptic systems with piecewise  $C^{m,\gamma}$  coefficients which come from composite materials. This answers the open problem suggested by Li and Vogelius in 2000 and Li and Nirenberg in 2003."

- 10:35 - 11:05 문병수 교수 (인천대)

Title: On a shallow-water model with the Coriolis effect

Abstract: In this talk an asymptotic model for wave propagation in shallow water with the effect of the Coriolis force is derived from the governing equation in two dimensional flows. The transport equation theory is then applied to investigate the local well-posedness and wave breaking phenomena for this model. The nonexistence of the Camassa-Holm-type peaked solution and classification of various traveling-wave solutions to the new system are also established. Moreover it is shown that all the symmetric waves to this model are traveling waves.

- 11:10 - 11:40 서이혁 교수 (성균관대)

Title: On the radius of spatial analyticity for the Klein-Gordon-Schrödinger system

Abstract: In this talk I shall consider the Klein-Gordon-Schrödinger system. This system is a classical model which describes a system of complex scalar nucleon fields interacting with neutral real scalar meson fields. The well-posedness of the system with initial data in Sobolev spaces has been intensively studied. Once we have the well-posedness, it is often of great interest whether spatial analyticity of the initial data persists at later times. More precisely, if the initial data are real-analytic and have a uniform radius of analyticity, so there is a holomorphic extension of the data to a complex strip, then we may ask whether or not and up to what degree the solution at some later time preserves the initial analyticity; we would like to estimate the radius of analyticity of the solution at later time, which is possibly shrinking. This talk is based on the recent joint work with Jaeseop Ahn and Jimyeong Kim.

- 13:30 - 14:00 광철광 교수 (이화여대)

Title: Dynamics of zero-speed solutions for Camassa-Holm type equations

Abstract: In this talk, we consider globally defined solutions of Camassa-Holm type equations outside the well-known non-zero speed, peakon region. These equations include the standard Camassa-Holm, Degasperis-Procesi equations, as well as nonintegrable generalizations such as the b-family, elastic rod, and Benjamin-Bona-Mahony equations. We are going to particularly see that, under the suitable assumptions, zero-speed and breather solutions do not exist at least in a spatial interval of appropriate size. As a consequence, we also see scattering and decay in Camassa-Holm type equations with long range nonlinearities.

- 14:05 - 14:35 석진명 교수 (경기대)

Title: Uniqueness of McCann's binary star

Abstract: Since a ground breaking work by Cazenave-Lions in 1982, showing uniqueness (up to symmetries) of variationally constructed solutions to Hamiltonian PDEs has played an indispensable role for verifying their orbital stability. In this talk, we discuss how to obtain the uniqueness of a family of binary star solutions to the Euler-Poisson equations, variationally constructed by McCann in 2006. Main methodology is based on perturbation arguments crucially relying on the exact asymptotic behaviors of solutions.

- 14:55 - 15:25 옥지훈 교수 (서강대)

Title: Local Hölder regularity for nonlocal equations with variable powers

Abstract: In this talk, we discuss on nonlocal problems with variational setting and present the local boundedness and the local Hölder continuity of weak solutions to nonlocal equations with variable orders and exponents under sharp assumptions.

- 15:30 - 16:00 진상돈 박사 (중앙대)

Title: Orbital stability for the mass-(super)critical pseudo-relativistic NLS

Abstract: For the one-dimensional mass-critical/supercritical pseudo-relativistic nonlinear Schrodinger equation, a stationary solution can be constructed as an energy minimizer under an additional kinetic energy constraint, and the orbital stability of the set of energy minimizers is well-known. In this work, by proving its local uniqueness, we establish the orbital stability of the solitary wave improving that of the energy minimizer set. A key aspect is reformulation of the variational problem in the non-relativistic regime, which is, we think, more natural because the proof heavily relies on the subcritical nature of the limiting model. This work is based on joint work with Younghun Hong.